Polarized Electromagnetic Radiation from Spatially Correlated Sources

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Abstract

We consider the effect of spatial correlations on sources of polarized electromagnetic radiation. The sources, assumed to be monochromatic, are constructed out of dipoles aligned along a line such that their orientation is correlated with their position. In one representative example, the dipole orientations are prescribed by a generalized form of the standard von Mises distribution for angular variables such that the azimuthal angle of dipoles is correlated with their position. In another example the tip of the dipole vector traces a helix around the symmetry axis of the source, thereby modelling the DNA molecule. We study the polarization properties of the radiation emitted from such sources in the radiation zone. For certain ranges of the parameters we find a rather striking angular dependence of polarization. This may find useful applications in certain biological systems as well as in astrophysical sources.

1 Motivation

In a series of interesting papers Wolf [1, 2, 3] studied the spectrum of light from spatially correlated sources and found, remarkably, that in general the spectrum does not remain invariant under propagation even through vacuum. The phenomenon was later confirmed experimentally [4, 5, 6] and has been a subject of considerable interest [7]. Further investigations of the source correlation effects have been done in the time domain theoretically [8] and experimentally [9]. Several applications of the effect have also been proposed [10, 11, 12, 13, 14]. In a related development it has been pointed out that spectral changes also arise due to static scattering [15, 16, 17, 18, 19] and dynamic scattering [20, 21, 22, 23, 24]. In the present paper we investigate polarization properties of spatially correlated sources. Just as we expect spectral shifts for such sources, we expect nontrivial polarization effects if the correlated source emits polarized light.

The basic idea can be illustrated by considering two polarized point sources P_1 and P_2 which are located along the z-axis at a distance $2z$ apart, in analogy to a similar situation considered by Wolf [2] for the unpolarized case. The electric field at the point Q located at large distances R_1 and R_2 from these point sources can be written as,

$$\vec{E} = \vec{e}(\vec{r}_1,\omega)\frac{e^{ikR_1}}{R_1} + \vec{e}(\vec{r}_2,\omega)\frac{e^{ikR_2}}{R_2} \tag{1}$$

Here $\vec{r}_1$ and $\vec{r}_2$ are the position vectors of the points P_1 and P_2. We calculate the coherency matrix for this electric field at the point Q,

$$\begin{aligned} < E_i^* E_j > \;&=\; \frac{< e_i^*(\vec{r}_1,\omega) e_j(\vec{r}_1,\omega) >}{R_1^2} + \frac{< e_i^*(\vec{r}_2,\omega) e_j(\vec{r}_2,\omega) >}{R_2^2} \\ &+\; \left(\frac{< e_i^*(\vec{r}_1,\omega) e_j(\vec{r}_2,\omega) > e^{ik(R_2-R_1)}}{R_1 R_2} + h.c.\right) \end{aligned} \tag{2}$$

We are interested in investigating the effect of the third term which arises due to cross correlation between the sources P_1 and P_2. It is clear that this term will give nontrivial contribution to the polarization in the far zone. In the present paper we investigate the contribution of this term by considering only monochromatic waves. It turns out to be useful to understand this simple idealization before treating the realistic case of partially polarized radiation.

In order to illustrate the contribution of the cross correlation term we consider the situation where the two sources are simple dipoles, $\vec{p}_1$ and $\vec{p}_2$ located at z and $-z$ respectively and are oriented such that their

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polar angles $\theta_1 = \theta_2 = \theta_p$ and the azimuthal angles $\phi_1 = -\phi_2 = \pi/2$. We assume that θ_p lies between 0 and $\pi/2$. The strength of the dipoles is p_0 and they radiate at frequency ω. We compute the electric field at point Q located at coordinates (R, θ, ϕ), as shown in Fig. 1, such that $R >> z$. The electric field in the far zone is obtained by the addition of two vectors $\hat{p}_1 \cdot \hat{R}\hat{R} - \hat{p}_1$ and $\hat{p}_2 \cdot \hat{R}\hat{R} - \hat{p}_2$ with phase difference of $2\omega z \cos\theta/c$. The vector $\hat{p} \cdot \hat{R}\hat{R} - \hat{p}$ at any point (R, θ, ϕ) is ofcourse simply the projection of the polarization vector $\hat{p}$ on the plane perpendicular to $\hat{R}$ at that point. We keep only the leading order terms in z/R in computing the total electric field.

The observed polarization is obtained by calculating the coherency matrix, given by

$$J = \begin{pmatrix} \langle E_\theta E_\theta^* \rangle & \langle E_\theta E_\phi^* \rangle \\ \langle E_\phi^* E_\theta \rangle & \langle E_\phi E_\phi^* \rangle \end{pmatrix} \tag{3}$$

The state of polarization can be uniquely specified by the Stokes's parameters or equivalently the Poincare sphere variables [28]. The Stoke's parameters are obtained in terms of the elements of the coherency matrix as: $S_0 = J_{11} + J_{22}$, $S_1 = J_{11} - J_{22}$, $S_2 = J_{12} + J_{21}$, $S_3 = i(J_{21} - J_{12})$. The parameter S_0 is proportional to the intensity of the beam. The Poincare sphere is charted by the angular variables 2χ, and 2ψ, which can be expressed as:

$$S_1 = S_0 \cos 2\chi \cos 2\psi, \quad S_2 = S_0 \cos 2\chi \sin 2\psi, \quad S_3 = S_0 \sin 2\chi \tag{4}$$

The angle χ $(-\pi/4 \leq \chi \leq \pi/4)$ measures of the ellipticity of the state of polarization and ψ $(0 \leq \psi < \pi)$ measures alignment of the linear polarization. For example, $\chi = 0$ represents pure linear polarization and $\chi = \pi/4$ pure right circular polarization.

The Stokes parameters at the observation point Q for the two dipole source are given by

$$S_0 = \left(\frac{\omega^2 p_0}{c^2 R}\right)^2 \left[4\cos^2(\omega z \cos\theta/c)\cos^2\theta_p \sin^2\theta + 4\sin^2(\omega z\cos\theta/c)\sin^2\theta_p(\cos^2\theta\sin^2\phi + \cos^2\phi)\right]$$

$$S_1 = \left(\frac{\omega^2 p_0}{c^2 R}\right)^2 \left[4\cos^2(\omega z \cos\theta/c)\cos^2\theta_p \sin^2\theta + 4\sin^2(\omega z\cos\theta/c)\sin^2\theta_p(\cos^2\theta\sin^2\phi - \cos^2\phi)\right]$$

$$S_2 = \left(\frac{\omega^2 p_0}{c^2 R}\right)^2 8\sin^2(\omega z\cos\theta/c)\sin^2\theta_p \cos\theta\sin\phi\cos\phi$$

$$S_3 = \left(\frac{\omega^2 p_0}{c^2 R}\right)^2 8\cos(\omega z\cos\theta/c)\sin(\omega z\cos\theta/c)\cos\theta_p\sin\theta_p\sin\theta\cos\phi$$

The resulting polarization in the far zone is quite interesting. At $\sin\phi = 0$, $\theta = \pi/2$ the wave is linearly polarized $(\chi = 0)$ with $\psi = 0$. As θ decreases from $\pi/2$ to 0, $\chi > 0$ and the wave has general elliptical polarization with $\psi = 0$. At a certain value of the polar angle $\theta = \theta_t$ the wave is purely right circularly polarized. As θ crosses θ_t, the linearly polarized component jumps from 0 to $\pi/2$, i.e. 2ψ changes from 0 to π. The value of the polar angle θ_t at which the transition occurs is determined by

$$\tan(\omega z\cos\theta_t/c) = \pm\sin\theta_t/\tan\theta_p \tag{5}$$

From this equation we see that as $z \to 0$, the transition angle θ_t is close to zero for a wide range of values of θ_p. Only when $\theta_p \to \pi/2$, a solution with θ_t significantly different from 0 can be found. In general, however, we can find a solution with any value of θ_t by appropriately adjusting z and θ_p. For $\sin\phi > 0$, we find qualitatively similar results, except that now the linear polarization angle 2ψ increases smoothly from 0 as θ goes from $\pi/2$ to 0. Furthermore the state of polarization never becomes purely circular, i.e. the angle 2χ is never equal to $\pi/2$ although it does achieve a maxima at θ close to the transition angle θ_t given by Eq. 5.

We therefore find that for wide range of parameters the polarization in the far field region shows a nontrivial dependence on angular position which is governed by the cross correlation term. In general this term will give nontrivial contribution unless the two sources have identical polarizations.

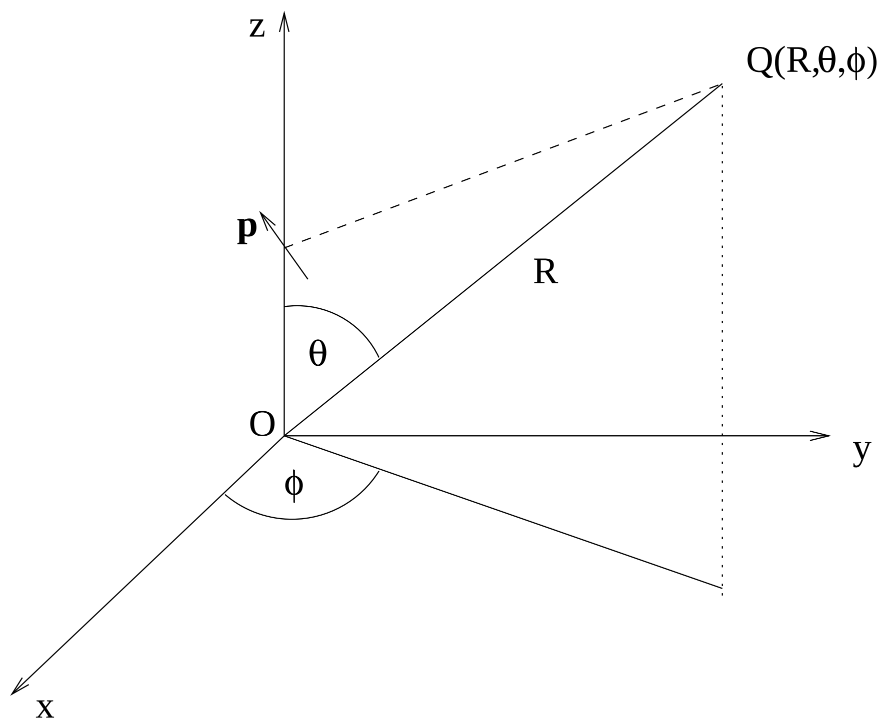

Figure 1: The correlated source consisting of an array of dipoles aligned along the z axis. The observation point Q is at a distance R which is much larger than the spatial extent of the source.

2 A One Dimensional Spatially Correlated Source

A simple continuous model of a monochromatic spatially correlated source can be constructed by arranging a series of dipoles along a line with their orientations correlated with the position of the source. The dipoles will be taken to be aligned along the z axis and distributed as a gaussian $\exp[-z^2/2\sigma^2]$. The orientation of the dipole is characterized by the polar coordinates θ_p, ϕ_p, which are also assumed to be correlated with the position z. A simple correlated ansatz is given by

$$\frac{\exp\left[\alpha\cos(\theta_p) + \beta z\sin(\phi_p)\right]}{N_1(\alpha)N_2(\beta z)} \tag{6}$$

where α and β are parameters, $N_1(\alpha) = \pi I_0(\alpha)$ and $N_2(\beta z) = 2\pi I_0(\beta z)$ are normalization factors and I_0 is the Bessel function. The basic distribution function $\exp(\alpha\cos(\theta - \theta_0))$ used in the above ansatz is the well known von Mises distribution which for circular data is in many ways the analoque of Gaussian distribution for linear data [25, 26, 27]. For $\alpha > 0$ this function peaks at $\theta = \theta_0$. Making a Taylor expansion close to its peak we find a gaussian distribution to leading power in $\theta - \theta_0$. The maximum likelihood estimators for the mean angle θ_0 and the width parameter α are given by, $< \sin(\theta - \theta_0) >= 0$ and $< \cos(\theta - \theta_0) >= d\log(I_0(\alpha))/d\alpha$ respectively. In prescribing the ansatz given in Eq. 6 we have assumed that the polar angle θ_p of the dipole orientation is uncorrelated with z and the distribution is peaked either at $\theta_p = 0$ (π) for $\alpha > 0$ (< 0). The azimuthal angle ϕ_p is correlated with z such that for $\beta > 0$ and $z > 0(< 0)$ the distribution peaks at $\phi_p = \pi/2(3\pi/2)$.

We next calculate the electric field at very large distance from such a correlated source. The observation point Q is located at the position (R, θ, ϕ) (Fig 1), measured in terms of the spherical polar coordinates, and we assume that the spatial extent of the source $\sigma << R$. The electric field from such a correlated source at large distances is given by,

$$E = -\frac{\omega^2}{c^2 R} p_0 e^{i(-\omega t + R\omega/c)} \int_{-\infty}^{\infty} dz \exp\left(-z^2/2\sigma^2\right) \int_0^{\pi} d\theta_p \int_0^{2\pi} d\phi_p$$

$$\times \quad \frac{\exp\left(\alpha\cos\theta_p + \beta z \sin\phi_p\right)}{2\pi^2 I_0(\alpha) I_0(\beta z)} \exp\left(i\omega z \hat{R}\cdot\hat{z}/c\right) \times \left(\hat{p}\cdot\hat{R}\hat{R} - \hat{p}\right) \tag{7}$$

where $\hat{p}$ is a unit vector parallel to the dipole axis, p_0 is the strength of the dipole, ω is the frequency of light and I_0 denotes the Bessel function. Since we are interested in the radiation zone we have dropped all terms higher order in z/R. The resulting field is ofcourse transverse i.e. $\vec{E}\cdot\hat{R} = 0$. We have also assumed that all the dipoles radiate at same frequency and are in phase. The correlation of the source with position is measured by the parameter β.

It is convenient to define scaled variable $\overline{z} = z/\sigma$, $\overline{\lambda} = \lambda/\sigma$ where $\lambda = 2\pi\omega/c$ is the wavelength, and $\overline{\beta} = \beta\sigma$. The integrations over θ_p and ϕ_p can be performed analytically. We numerically integrate over z for various values of position of the observation point, the parameter α which determines the width of the distribution of θ_p and for different value of the correlation parameter β.

We first study the situation where $\overline{\beta} > 0$ and $\alpha > 0$. The result for several values of (θ, ϕ) are given in figures 2,3 which show plots of the Poincare sphere variables 2χ and 2ψ. The scaled wavelength $\overline{\lambda} = \lambda/\sigma$ of the emitted radiation is taken to be equal to π, i.e. the effective size of the source σ is of the order of the wavelength λ. The results show several interesting aspects. The ellipticity of the state of polarization shows significant dependence on the position of the observer. The angle $\chi = 0$, i.e. the beam has pure linear polarization, for the polar angle $\cos(\theta) = 0, 1$ for all values of azimuthal angle ϕ. It deviates significantly from 0 as $\cos(\theta)$ varies from 0 to 1. For $\sin(\phi) = 0$, $2\chi = \pi/2$ at some critical value θ_t as $\cos(\theta)$ varies between 0 and $\pi/2$, i.e. the state of polarization is purely right circular at $\theta = \theta_t$. For $\sin(\phi) > 0$, 2χ also deviates significantly from 0 and displays a peak at some value of θ. The precise position of the peak is determined by the values of the correlation parameters α and $\overline{\beta}$.

The alignment of linear polarization also shows some very interesting aspects. For $\sin(\phi) = 0$, we find that ψ is either 0 or π depending on the value of θ. The transition occurs at the same critical value of θ where the angle χ shows a peak. The state of polarization is purely linear with the electric field along the $\hat{\theta}$ for $\cos(\theta) = 0$ and then acquires a circular component for increasing values of $\cos(\theta)$. At the transition point $\theta = \theta_t$, the polarization is purely circular. With further increase in value of θ the state of polarization is elliptical with the linearly polarized component aligned along $\hat{\phi}$. The transition point is clearly determined by the condition $S_1 = J_{11} - J_{22} = 0$.

For other values of $\sin(\phi)$ we find $\psi = 0$ for $\cos\theta = 0$ and then deviates significantly from 0 as θ approached θ_t, finally levelling off as $\cos\theta$ approches 1. The final value of ψ at $\cos\theta = 1$ depends on the correlation parameters and $\sin\phi$ but for a wide range of parameters $2\psi > \pi/2$. We see from Fig. 3 that the linear polarizations from sources of this type shows striking characteristic, i.e. that the polarization angle ψ is either close to 0 or $\pi/2$ depending on the angle at which it is viewed.

For $\sin\phi < 0$ the Poincare sphere polar angle 2χ is same as for $\sin\phi > 0$, however the orientation of the linear polarization 2ψ lies between π and 2π, i.e. in the third and fourth quadrants of the equatorial plane on the Poincare sphere. For a particular value of ϕ the azimuthal angle $\psi(\phi) = -\psi(-\phi)$.

If we change the sign of α we do not find any change in linear polarization angle ψ however the value of χ changes sign, i.e. the state of polarization changes from right elliptical to left elliptical. Change in sign of $\overline{\beta}$ also leaves ψ unchanged while changing the sign of χ. Changing the signs of both α and $\overline{\beta}$ produces no change at all.

In the case of the limiting situation where $\overline{\beta} = 0$ we find, as expected, linear polarization is independent of the angular position, i.e. $\chi = 0$ and $\psi = 0$. This is true for any value of the parameter α, which determines the polar distribution of the dipole orientations. Hence we see that the effect disappears if either the effective size of the source $\sigma = 0$ or the correlation parameter $\beta = 0$. The effect also dissappears in the limit $\alpha \to \infty$. In this limit the distribution of θ_p is simply a delta function peaked at 0 and hence our model reduces to a series of dipoles aligned along the z-axis, which cannot give rise to any nontrivial structure. In the numerical calculations above we have taken the effective size of the source σ of the order of the wavelength λ. If the size $\sigma << \lambda$, the effect is again negligible since the phase factor $\omega z \hat{R}\cdot\hat{z}/c$ in Eq. 7 is much smaller than one in this case.

Hence we find that in order to obtain a nontrivial angular dependence of the state of polarization the size of the source, assumed to be coherent, has to be of the order of or larger than the wavelength as well as the correlation length $1/\beta$.

2.1 Transition angle

From our results we see that there exists a critical value of the polar angle θ at which the state of linear polarization changes very rapidly. This is particularly true if we set $\sin\phi = 0$ where we find that that the orientation of linear polarization ψ suddenly jumps from 0 (or π) to $\pi/2$ at some critical value of the polar angle $\theta = \theta_t$. We study this case in a little more detail. The θ and ϕ components of the total electric field is given by,

$$E_\theta = -\frac{\omega^2 p_0}{c^2 R} e^{-i\omega(t-R/c)} \sqrt{2\sigma^2\pi} e^{-\sigma^2\omega^2\cos^2\theta/2c^2} \frac{I_1(\alpha)}{I_0(\alpha)} \sin\theta$$

$$E_\phi = i\frac{\omega^2 p_0}{c^2 R} e^{-i\omega(t-R/c)} \frac{2\sinh\alpha}{\alpha\pi I_0(\alpha)} A$$

$$A = \int_{-\infty}^{\infty} dz e^{-z^2/2\sigma^2} \sin(\omega z\cos\theta/c) \frac{I_1(\beta z)}{I_0(\beta z)}$$

In this case the Stokes parameter $S_2 = 0$. For $\beta \geq (<) 0$, $S_3 \geq (<) 0$ and hence $\chi \geq (<) 0$. The point where the polarization angle 2ψ jumps from 0 to π is determined by the condition $S_1 = 0$. This is clearly also the point where $2\chi = \pm\pi/2$. Explicitly the condition to determine the critical value θ_t is,

$$A^2 = \sigma^2\pi^3\alpha^2 e^{-\sigma^2\omega^2\cos^2\theta_t/c^2} \sin^2\theta_t \frac{I_1(\alpha)^2}{2\sinh^2\alpha}.$$

This can be used to determine θ_t as a function of α, β. The result for $\cos\theta_t$ as a function of α is plotted in figure 4 for several different values of β. For any fixed value of the parameter β, the transition angle θ_t decreases from $\pi/2$ to 0 as α goes from zero to infinity. This is expected since as α becomes large the polar angle distribution of the dipole orientations, peaked along the z axis, becomes very narrow and hence the resultant electric field is aligned along the z axis for a large range of polar angle θ. Furthermore we find, as expected, that as β goes to zero the transition angle also tends towards 0.

3 Helical Model

We next study an interesting generalization of the model discussed above. Instead of the having the peak of the ϕ_p distribution fixed to $-\pi/2$ for $z < 0$ and $\pi/2$ for $z > 0$ we allow it to rotate in a helix circling around the z-axis. In this case we replace the ϕ_p dependence by $\exp[\beta(\phi_p - \xi z)]$. As z goes from negative to positive values, the peak of the distribution rotates clockwise around the z-axis forming an helix. This is a reasonable model of the structure of DNA molecule and hence has direct physical applications. We study this in detail by fixing the azimuthal angle of the dipole orientation $\phi_p = \xi z$ and the polar angle θ_p to some constant value, i.e. the ϕ_p and θ_p distributions are both assumed to be delta functions. This allows us to perform the z integration in Eq. 7 analytically. The resulting state of polarization, described by Poincare sphere angles 2χ and 2ψ are shown in Figs. 5-8 . In this model we can extract a simple rule to determine the transition angle for the special case $\theta_p = \pi/2$ and $\xi = n\pi$ where n is an integer. We set $\sin\phi = 0$ for this calculation since it is only for this value that the polarization becomes purely circular for some value of $\theta = \theta_t$ and the linearly polarized component flips by $\pi/2$ at this point. A straightforward calculation shows that this transition angle θ_t is given by:

$$\cos^2\theta_t = n\lambda/2$$

Here n represents the number of π radians that are traversed by the tip of electric field vector along the helical path and λ is the wavelength. In order to get at least one transition $\lambda < 2/n$. In the special case under consideration there is atmost one transition. However in general the situation is more complicated and for certain values of θ_p and ξ, more than one transitions are possible. Some representative examples are shown in Figs. 5-8.

4 Conclusions

In this paper we have considered spatially correlated monochromatic sources. We find that at large distance the polarization of the wave shows dramatic dependence on the angular position of the observer. For certain set of parameters the linearly polarized component shows a sudden jump by $\pi/2$. If the symmetry axis of the source is taken to be the z-axis, the polarization shows a sudden transition from being parallel to perpendicular to the symmetry axis of the source, as the polar angle is changed from $\pi/2$ to 0. The sources considered in this paper are idealized since we have assumed coherence over the entire source. For small enough sources, such as the DNA molecule, this may a reasonable approximation. In the case of macroscopic sources, this assumption is in general not applicable. However in certain situations some aspects of the behavior described in this paper may survive even for these cases. For example, we may consider a macroscopic source consisting of large number of structures of the type considered in this paper. As long as there is some correlation between the orientation of these structures over large distances we expect that some aspects of the angular dependence of the polarization of the small structures will survive, even if there does not exist any coherent phase relationship over large distances. Hence the ideas discussed in this paper may also find interesting applications to macroscopic and astrophysical sources. As an interesting example we consider astrophysical sources of radio waves. It is well known that the polarization angle of these sources is predominantly observed to be aligned either parallel or perpendicular to the source orientation axis [29]. This difference has generally been attributed to the existence of different physical mechanism for the generation of radio waves in these sources. Our study, however, indicates that this difference in observed polarization angle could also arise simply due to different angles of observation. Hence orientation effects must be considered before attributing different physical mechanisms for differences in observed polarizations of these sources.

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References

[1] E. Wolf, Phys. Rev. Lett. **56**, 1370 (1986).

[2] E. Wolf, Optics Communication **62**, 12 (1987).

[3] E. Wolf, Nature **326**, 26 (1987).

[4] D. Faklis and G. M. Morris, Opt. Lett. **13**, 4 (1988).

[5] F. Gori, G. Guattari, C. Palma and G. Padovani, Opt. Commun. **67**, 1 1988).

[6] G. Indebetouw, J. Mod. Phys. **36**, 251 (1989).

[7] For a review see E. Wolf and D. F. V. James, Rep. Progr. Phys. **59**, 771 (1996).

[8] J. Rai, S. Rai and S. Chopra, Phys. Rev. A **47**, 4400 (1993).

[9] S. Chopra, S. K. P. Bhat, J. Joseph, S. Rai and J. Rai, Opt. Commum. **109**, 205 (1994).

[10] D. F. V. James, H. C. Kandpal and E. Wolf, Astrophys. J. **445**, 406 (1995).

[11] H. C. Kandpal, K. Saxena, D. S. Mehta, J. S. Vaishya and K. C. Joshi, J. Mod. Opt. **42**, 447 (1995).

[12] S. Vicalvi, G. Shirripa Spagnolo, and M. Santarsiero, Opt. Commun. **130**, 241 (1996).

[13] E. Wolf, T. Shirai, H. Chen and W. Wang, J. Mod. Opt. **44**, 1345 (1997).

[14] T. Shirai, E. Wolf, H. Chen and W. Wang, J. Mod. Opt. **45**, 799 (1998).

[15] E. Wolf, J. T. Foley and F. Gori, J. Opt. Soc. Am. A **6**, 1142 (1989); erratum, **7**, 173 (1990).

[16] T. Shirai and T. Asakura, J. Opt. Soc. Am. **12**, 1354 (1995).

[17] T. Shirai and T. Asakura, Opt. Commun. **123**, 234 (1996).

[18] T. A. Leskova, A. A. Maradudin, A. V. Shchegrov and E. R. Mendez, Phys. Rev. Lett. **79**, 1010 (1997).

[19] A. Dogriu and E. Wolf, Opt. Lett. **23**, 1340 (1998).

[20] E. Wolf, Phys. Rev. Lett. **63**, 2220 (1989).

[21] J. T. Foley and E. Wolf, Phys. Rev. A **40**, 588 (1989).

[22] D. F. V. James, M. P. Savedoff and E. Wolf, Astrophys. J. **359**, 67 (1990).

[23] D. F. V. James and E. Wolf, Phys. Lett. A **146**, 167 (1990).

[24] D. F. V. James and E. Wolf, Phys. Lett. A **188**, 239 (1994).

[25] K. V. Mardia, *Statistics of Directional Data* (Academic Press, London, 1972).

[26] E. Batschelet, *Circular Statistics in Biology*, (London: Academic Press, 1981).

[27] N. I. Fisher, *Statistics of Circular Data*, (Cambridge, 1993).

[28] M. Born and E. Wolf, *Principles of Optics* (1980), Pergamon Press.

[29] J. N. Clark *et al*, Mon. Not. R. Astron. Soc. **190**, 205 (1980).

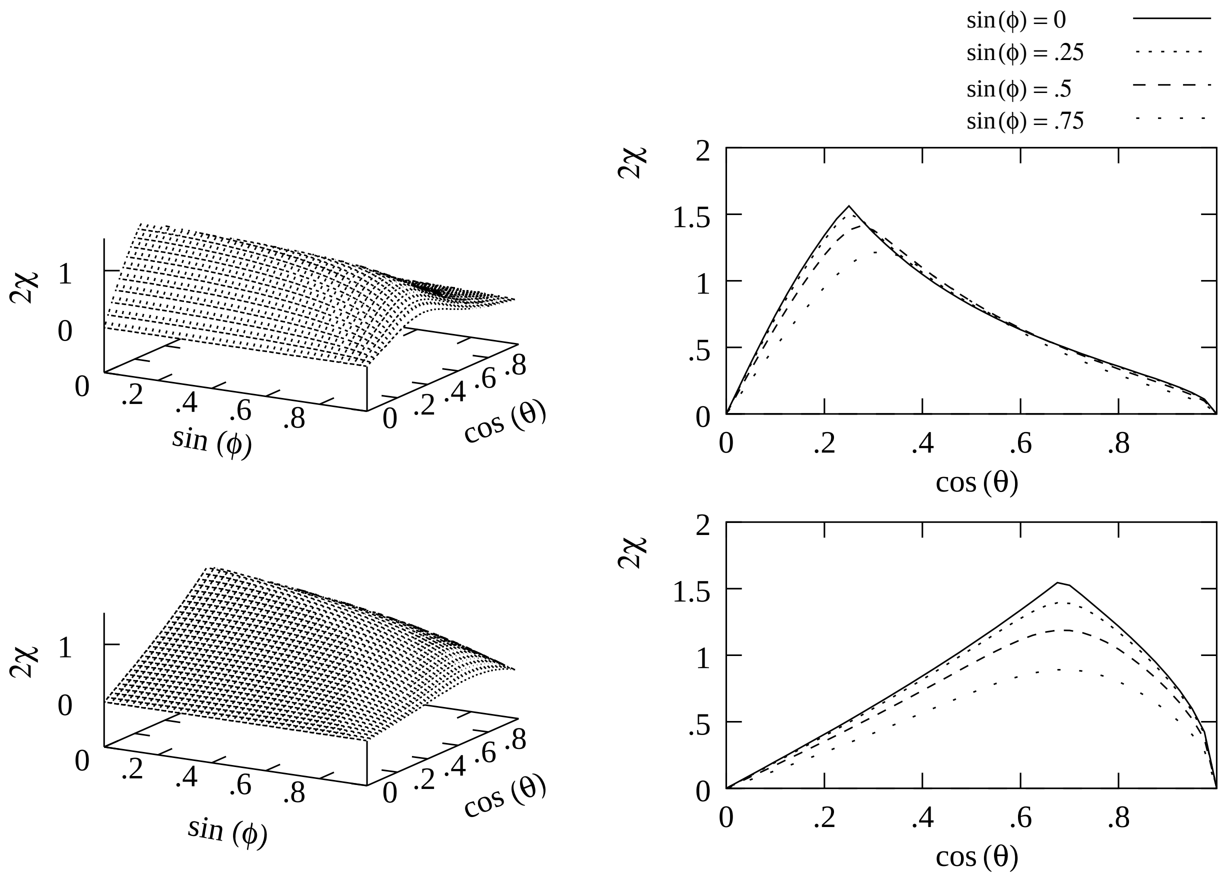

Figure 2: The polar angle on the Poincare sphere 2χ, which is a measure of the eccentricity of the ellipse traced by the electric field vector. For pure linear polarization $2\chi = 0$ and for pure right circular polarization $2\chi = \pi/2$. The 3-D plot shows 2χ as a function of $\cos\theta$ and $\sin\psi$ where θ and ϕ are the polar and azimuthal angles of the point of observation. The 2-D plots on the right show the corresponding slices of the 3-D plots for different values of $\sin\phi$. The upper and lower plots correspond to $\overline{\beta} = 1$, $\alpha = 0.25$ and $\beta = \alpha = 1$ respectively.

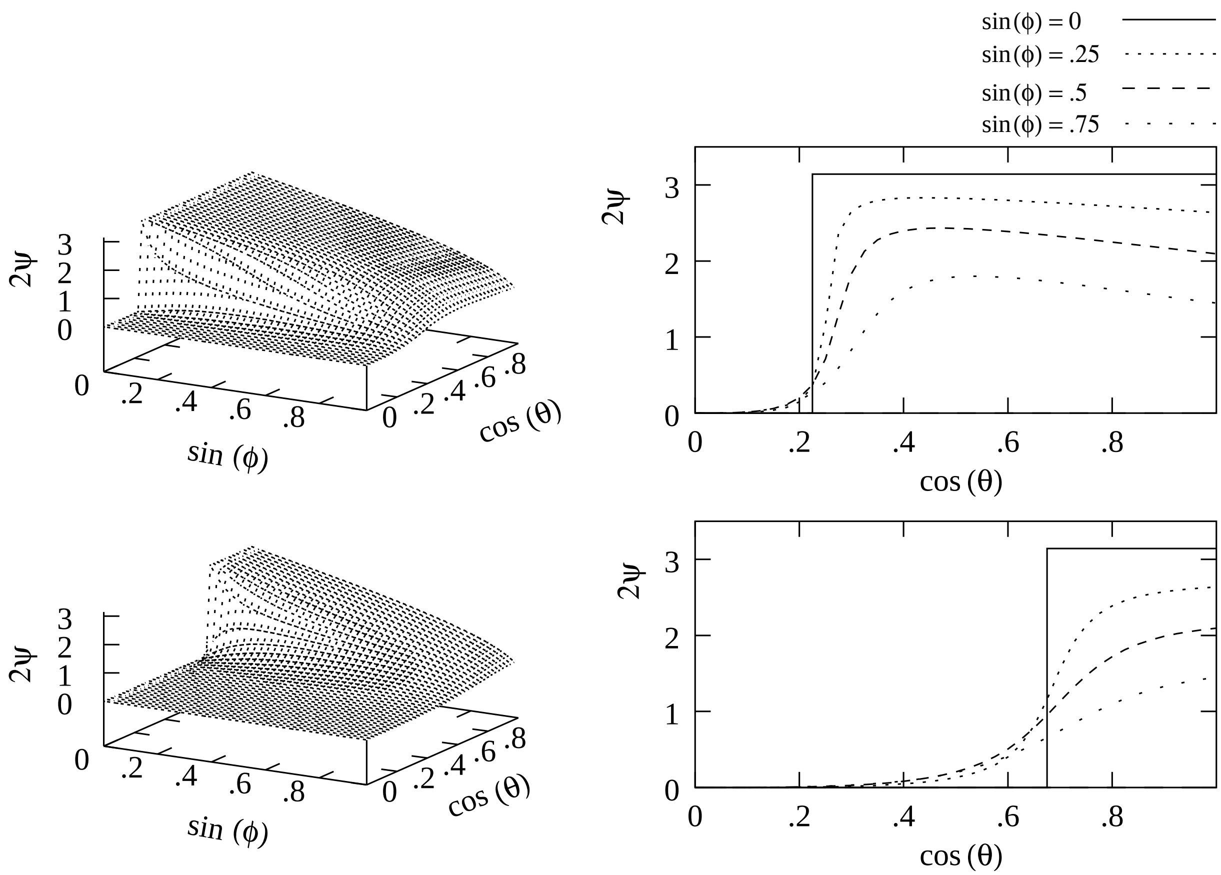

Figure 3: The azimuthal angle on the Poincare sphere 2ψ. This measures the orientation of the linearly polarized component of the wave. The 3-D plot shows 2ψ as a function of $\cos\theta$ and $\sin\psi$ where θ and ϕ are the polar and azimuthal angles of the point of observation. The 2-D plots on the right show the corresponding slices of the 3-D plots for different values of $\sin\phi$. The upper and lower plots correspond to $\overline{\beta} = 1$, $\alpha = 0.25$ and $\beta = \alpha = 1$ respectively.

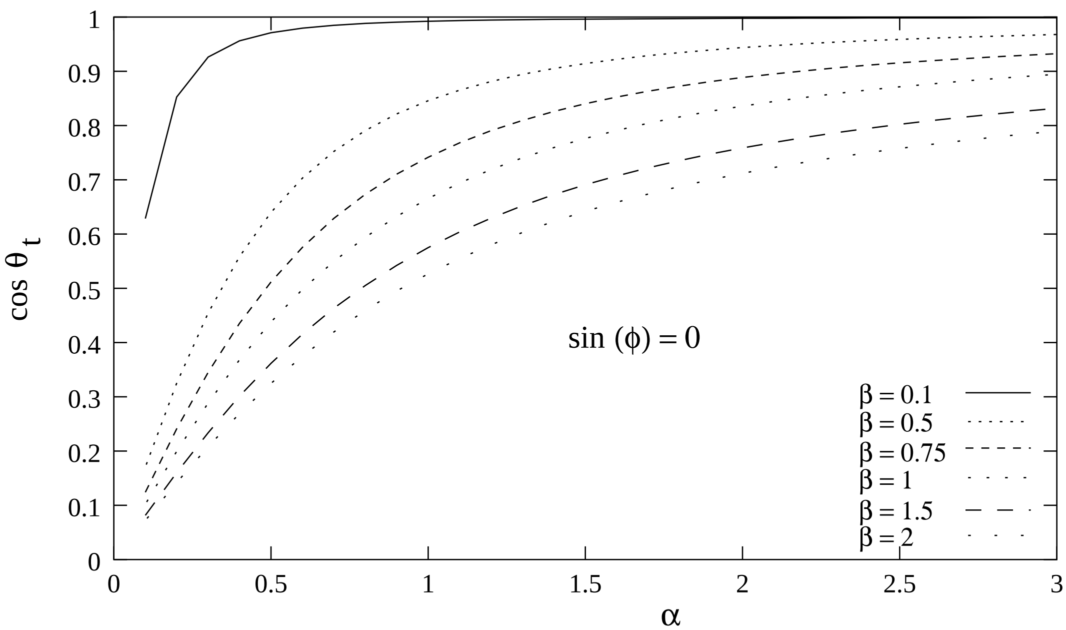

Figure 4: The critical value of the polar angle θ at which the state of linear polarization shows a sudden transition for $\sin\phi = 0$ as a function of the parameters α and β which specify the distribution of the dipole orientations. For any given value of the parameters α and β, electric field is parallel ($\psi = 0$) to z axis if the cosine of the observation polar angle $\cos\theta$ is less than $\cos\theta_t$. On the other hand electric field is perpendicular to the z axis if $\cos\theta$ is greater than $\cos\theta_t$.

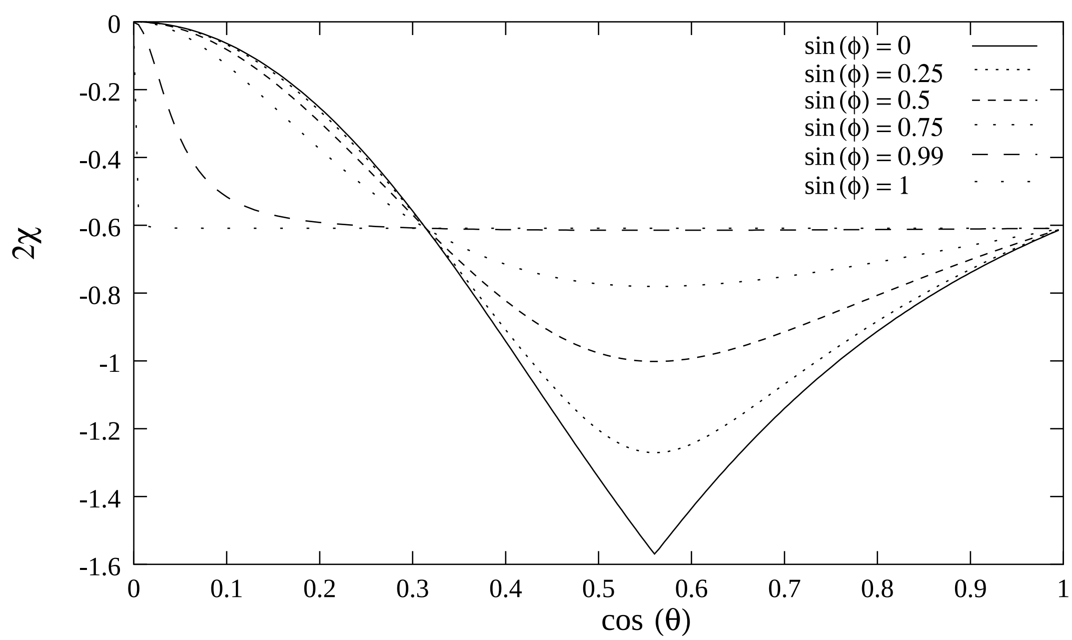

Figure 5: The polar angle on the Poincare sphere 2χ (radians) for the helical model as a function of cos(θ) ($\lambda = 0.2\pi, \theta_p = \pi/2, \xi = \pi$).

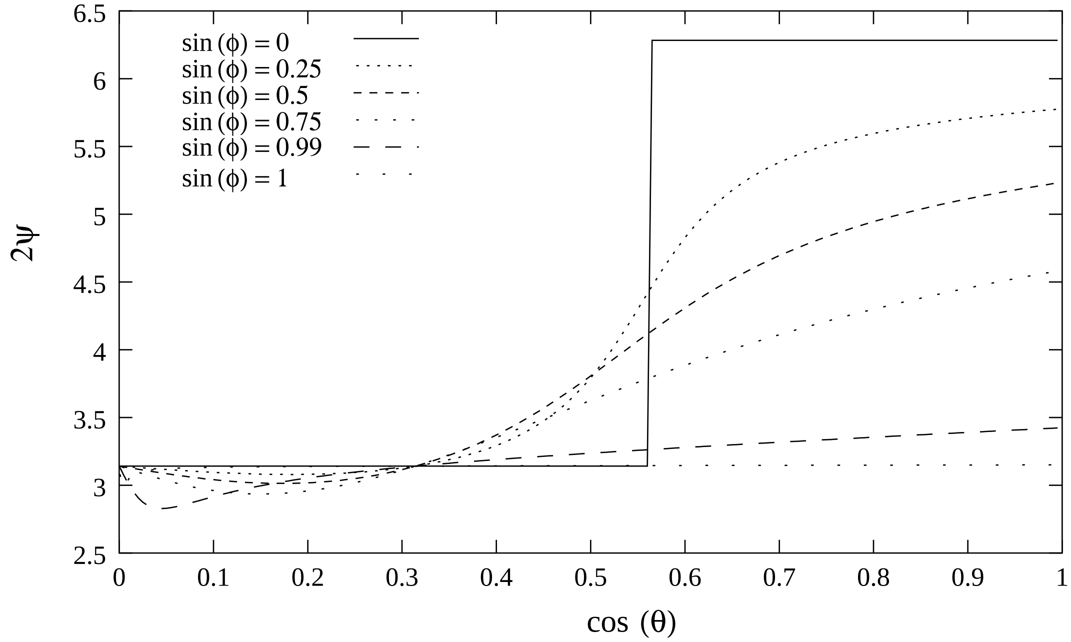

Figure 6: The azimuthal angle on the Poincare sphere 2ψ (radians) for the helical model as a function of $\cos(\theta)$ $(\lambda = 0.2\pi, \theta_p = \pi/2, \xi = \pi)$.

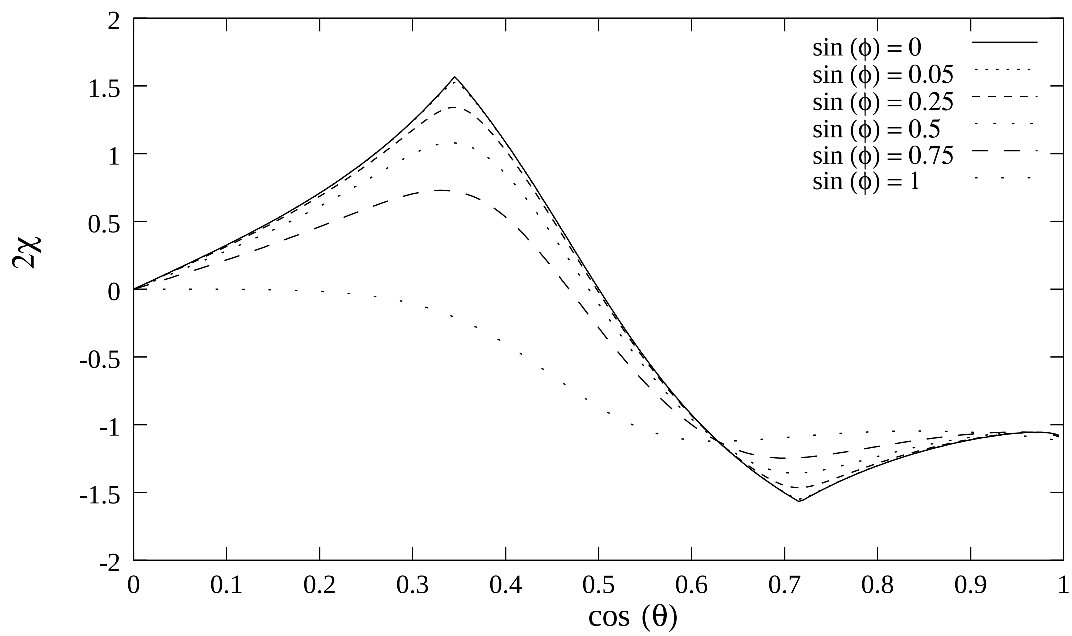

Figure 7: The polar angle on the Poincare sphere 2χ (radians) for the helical model as a function of $\cos(\theta)$ $(\lambda = 0.4\pi, \theta_p = \pi/4, \xi = \pi)$.

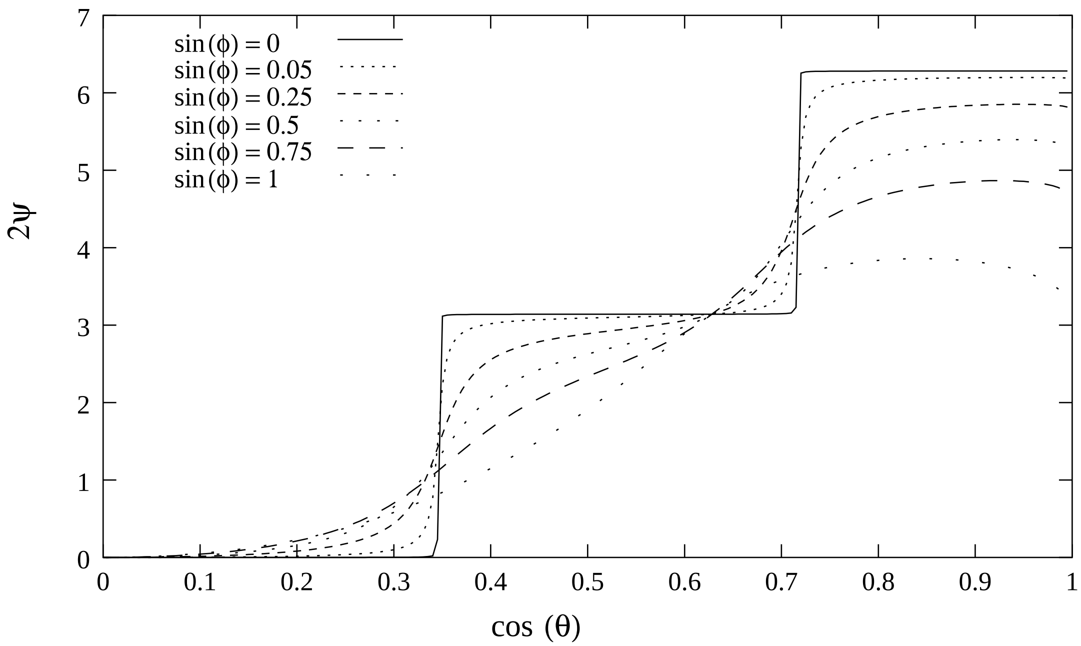

Figure 8: The azimuthal angle on the Poincare sphere 2ψ (radians) for the helical model as a function of $\cos(\theta)$ ($\lambda = 0.4\pi, \theta_p = \pi/4, \xi = \pi$).